Volume XCII, No. 1, May 2022

Proceedings of the Danish Society for Structural Science and Engineering

Published by

Danish Society for Structural Science and Engineering

COPENHAGEN 2022

ISSN 1601-6548 (online)

PROCEEDINGS OF THE DANISH SOCIETY FOR STRUCTURAL SCIENCE AND ENGINEERING
Edited and published by the Danish Society for Structural Science and Engineering
Volume 92, No. 1, 2022.

Frequency Based Damage Assessment of Bolted Specimen

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Abstract
This paper develops a vibration based 3-level damage identification scheme which aims to detect (level 1), localize (level 2) and quantify (level 3) single structural damages originated from stiffness changes. The damage identification scheme is applied on a bolted laboratory specimen made of aluminum and is performed purely by using the natural frequencies as the damage indicators. The scheme is built on the premise called Scenario Based Damage Assessment, where a finite element model representing the laboratory specimen is created. In addition, the critical failure mechanisms of the specimen are parametrized in the FE-model producing the hypothetical changes in frequencies which are stored in the sensitivity matrix. The 3-level damage identification scheme is performed on experimental tests. Here, loosening the torque of the bolts on the laboratory specimen represented experimental damages. Detection of damage proved to be possible on almost all damaged bolts using the proposed level 1 method where the coefficient of variance is determined. Moreover, both the level 2 and level 3 approaches proved to be competent methods able to localize correctly and estimate a sensible value of the damage extent.

1. Introduction

The application of structural health monitoring (SHM) is essential to maintain and conserve the service life of mechanical and civil structures in today's society. Particularly approaching the end of the limited-service life of structures, damages and material deterioration is indeed inevitable. As a result, several SHM techniques are currently being applied with sole purpose of prolonging the service life as well as saving repair costs and increasing performance efficiency.

SHM techniques can be classified into either local or global practices. Some local SHM applications include digital image correlation, ultrasonic testing, visual testing, and optical methods [1].

Damage assessment based on vibration characteristics establishes a branch within global SHM applications. Namely, vibration based SHM is rising to be a robust method to identify damages in structures by analyzing the global dynamic properties such as the natural frequencies, modeshapes and damping. A structural damage can be classified as an alteration in the material and geometric characteristics that fundamentally can affect the safety and performance [2] [3].

According to Rytter [4], a complete identification of a structural damage is composed of four different levels. The premier level is identifying the actual presence of damage. This level requires the least amount of

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information and can be performed relativity effortlessly by use of statistical measures. The next two levels are to determine the location and severity of the of detected damage. These levels demand considerable knowledge and information of the monitored structure, both in the undamaged and damaged state. The last level is to diagnose the damage and assess its impact on the remaining service life of the structure.

In summary, Rytter's four levels can be listed as follows.

Level 1 – Is the structure damaged?

Level 2 - Where is the damage?

Level 3 – How severe is the damage?

Level 4 – What is the remaining service life?

This paper presents a frequency-based damage assessment based on the first three Rytter levels. For each level an approach is presented. For Level 1, a statistical approach is introduced where the coefficient of variance of the natural frequencies is calculated in the undamaged state of the laboratory specimen. Damage is then detected, when the measured frequencies of damaged state exceed the threshold of calculated coefficients of variance. For Level 2 and Level 3, two approaches are presented that require a numerical model of the specimen, where hypothetical damage scenarios are parametrized. Here, the perturbed frequency changes are compared with the estimated frequency changes gathered from the parametric model. The comparison of the two entries is performed by two separate methods, one estimating the location of the perturbation and one yielding an approximated extent of the severity of the damage. The three approaches are all listed in Tabel 1*: - Damage Assessment Scheme*

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Level	**Classification**	**Approach**
Level 1	Detection	Coefficient of Variation
Level 2	Localization	MAC-SBDA
Level 3	Quantification	Pseudo-Inverse Method

Tabel 1: - Damage Assessment Scheme

Furthermore, this paper will concern structural damage as change in the stiffness. In the numerical model, the stiffness change will be defined as a reduction of the Young's-Modulus in the finite elements, whereas the experimental model will concern damage as stiffness loss in its bolts.

2. Scenario Based Damage Assessment

The main aspiration of SBDA is to compare modal parameters of hypothetical damage scenarios of a structure with the actual measured modal parameters. The hypothetical modal parameters of the scenarios are set up in a Finite Element (FE) model where each possible scenario is computed through modal analysis. In theory, an infinite number of damage locations can occur, however with SBDA, it is viable to narrow down the number of damage locations, where only the most critical positions are inspected. The SBDA is introduced in J.B Hansen's PhD dissertation [5]. However, this paper integrates the SBDA scheme with proposed frequency based 3-level damage identification. This integration can be explained by four steps as depicted in Figure 1.

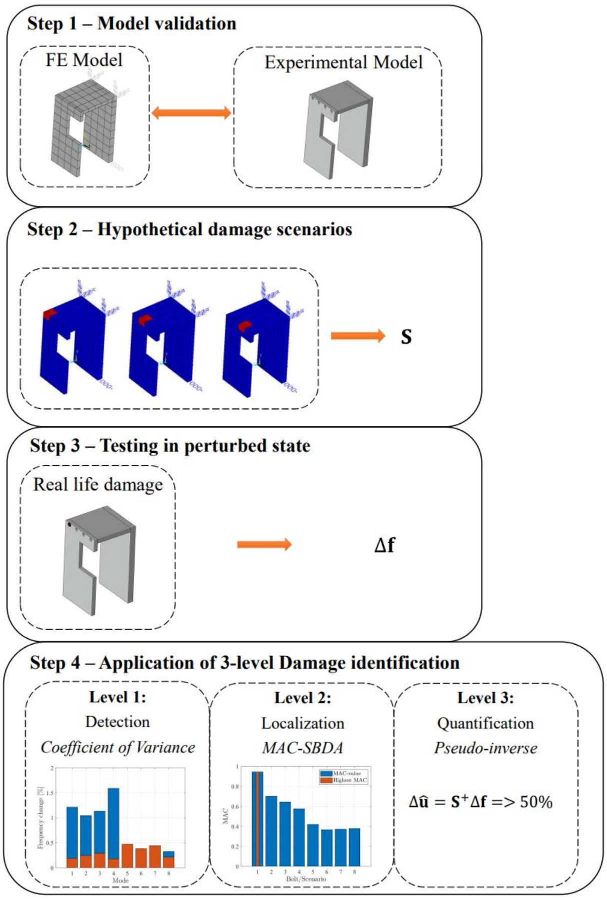


Figure 1: The four steps to perform the 3-level damage identification

The four steps presented in Figure 1 are explained in the following sections.

Step 1 – Model validation

The initial step of the SBDA protocol is to create a finite element model that represents the experimental model. The FE-model need to have the exact dimensions and material properties as the investigated specimen. Once an initial FE-model has been created, a modal analysis is performed to compute the mass-scaled modeshapes and natural frequencies. Subsequently experiments are conducted on the investigated specimen, where OMA is practiced extracting the modal parameters. Following this, the experimental modal parameters are used to validate the FE-model to assess the reliability of the model as a representation of the investigated specimen.

It must be noted that multiple tests shall be performed to extract the experimental frequencies in order to determine the coefficient of variance which will be used later in Step 4 for Level 1 - Damage Detection.

Step 2 – Hypothetical damage scenarios

Once the FE-model is designated to be a satisfactory representation of the specimen, the hypothetical damage scenarios are defined. In this project, the aluminum specimen is investigated and therefore the damage scenarios are narrowed down to eight which correspond to the number of bolts in the specimen. Each scenario is representing a loss of stiffness in an individual bolt. A modal analysis is then performed for each damaged case to compute the perturbed frequencies. The change in frequencies for each scenario is then stored in a sensitivity matrix denoted as $\mathbf{S}$. It must be noted that the sensitivity matrix is partially dependent on the accuracy of the FE-model.

Step 3 – Testing in perturbed state

Assuming damage has transpired, the specimen is tested in the same way as in Step 1. The frequencies from the perturbed state is gathered in order to determine the measured frequency changes. The frequency changes are finally stored in a column vector, $\Delta\mathbf{f}$.

Step 4 – Application of 3-level damage identification

The last step is to apply the 3-level damage identification. Firstly, the measured frequency changes of the perturbed state are plotted with calculated coefficient of variance. Once damage is clearly present, the MAC-SBDA is used to compare the measured frequency changes with the sensitivity matrix computed from the hypothetical scenarios to indicate the most probable location of the damage. The last stage is to utilize the pseudo-inverse approach to give an estimate of the damage extent. This extent is returned as an actual stiffness reduction.

3. Sensitivity of Natural Frequencies

When a damage in a structure occurs, the stiffness will be diminished, which results in the reduction of the natural frequencies. The change of natural frequency caused by stiffness reductions can be explained by different theoretical frameworks found in the literature [5] [6]. This paper utilizes a dimensionless formulation which was presented in the paper [7],

$$\frac{\omega_{ai}^2-\omega_{bi}^2}{\omega_{ai}^2}=\frac{1}{t_{ii}}\mathbf{b}_i^T(-\Delta\mathbf{M}+\Delta\mathbf{K}\frac{1}{\omega_{ai}^2})\mathbf{a}_i \tag{1}$$

Where $\mathbf{b}_i$ is the modeshape of the unperturbed system, $\mathbf{a}_i$ is the modeshape of the perturbed system, $\Delta\mathbf{M}$ is the change in the mass matrix and $\Delta\mathbf{K}$ is the change in the stiffness matrix. Subscript a and b symbolize the perturbed and unperturbed state of the system respectively.

Generally, it is quite difficult to attain accurate versions of mass and stiffness matrices using commercial finite element software, while it is easy to secure the modal parameters. Relation (1) gives an exact estimation of the right-hand side, which means by only the using left hand side, it will be sufficient when calculating the change in the natural frequencies for structural modifications affecting the stiffness and mass matrix.

In a scenario-based scheme, the frequency changes are utilized to determine the two entries, namely the sensitivity matrix $\mathbf{S}$ and the measured frequency change vector $\Delta\mathbf{f}$. As specified already, the sensitivity matrix stores the predicted frequency changes found by simulating hypothetical damage scenarios in a numerical model, thus having the dimensions $(N_m \times N_{sc})$.

$$\mathbf{s}_{\Delta\omega_j}=\begin{bmatrix}\Delta\omega_{j1}\\ \Delta\omega_{j2}\\ \vdots\\ \Delta\omega_{jNm}\end{bmatrix} \Rightarrow \mathbf{S}=\begin{bmatrix}\mathbf{s}_{\Delta\omega_1} & \mathbf{s}_{\Delta\omega_2} & ,.., & \mathbf{s}_{\Delta\omega_{Nsc}}\end{bmatrix} \tag{2}$$

Similarly for the measured frequency changes, the entries are contained in a column vector $\Delta\mathbf{f}$ with the dimensions $(N_m \times 1)$.

$$\Delta \mathbf{f} = \begin{bmatrix} \Delta\omega_1 \\ \Delta\omega_2 \\ \vdots \\ \Delta\omega_{Nm} \end{bmatrix} \tag{3}$$

3.1 MAC-SBDA

This section introduces a method for localization of structural perturbations in a scenario-based scheme. The approach is based on the original formulation of the modal assurance criteria (MAC) which is an established method within the field of structural dynamics and modal analysis. The method is named the MAC-SBDA was introduced in the paper [7]. However, a similar version of the MAC-SBDA is found in [8].

The MAC-SBDA deals with the two entries, namely the measured frequency change vector $\Delta\mathbf{f}$ and the sensitivity matrix $\mathbf{S}$. The MAC measures the linear correlation between the two entries such as the frequency changes for the scenario that best resembles the measured frequency change vector yield the highest value. The highest possible value is 1, whereas the lowest is 0. The MAC-SBDA is written in the following mathematical form,

$$\text{MAC}(\Delta\mathbf{f}, \mathbf{S}) = \frac{(\Delta\mathbf{f}^T\mathbf{S})^2}{(\Delta\mathbf{f}^T\Delta\mathbf{f})(\mathbf{S}^T\mathbf{S})} \tag{4}$$

3.2 Pseudo Inverse Method

An approach for quantifying a structural damage relates the measured frequency change vector $\Delta\mathbf{f}$ and the predicted frequency changes $\mathbf{S}$. The link between the two entries is presented in [5],

$$\Delta\mathbf{f} = \mathbf{S}\Delta\mathbf{u} \tag{5}$$

Isolating $\Delta\mathbf{u}$ the expression turns into,

$$\Delta\mathbf{u} = \mathbf{S}^{-1}\Delta\mathbf{f} \tag{6}$$

If the changes in frequencies between the predicted and measured changes are proportional to the perturbed matrices $\Delta\mathbf{M}$ and $\Delta\mathbf{K}$, $\Delta\mathbf{u}$ will compute which scenario the perturbation is occurring as well as the extent of the structural perturbation. In other words, $\Delta\mathbf{u}$ operates as an index vector which can localize and quantify a perturbation with respect to the parametrized scenarios. However, a well-conditioned and accurate sensitivity matrix must be constructed to successfully identify and quantify the exact perturbation.

When the sensitivity matrix $\mathbf{S}$ consists of more modes than number of predefined damage scenarios, then it is considered as over-determined. Therefore (6) is reformulated such as the Morse pseudo inverse of sensitivity matrix is determined.

$$\Delta\mathbf{u} = \mathbf{S}^{+}\Delta\mathbf{f} \tag{7}$$

Even though expression (7) covers both Level 2: localization and Level 3: quantification, the equation will solely be used as a damage quantification index. For Level 2 localization, the MAC will therefore be the proposed damage localizer.

4. Case Study of Bolted U-Specimen

In this section, the scenario-based damage assessment approach will be implemented on a bolted U-shaped specimen. The specimen consists of a three-aluminum plated bolted together by a total of eight bolts. Two of the three plates have a width of 100 mm and a height of 200 mm, where one the of plate have a reduced cross section and a different thickness. This makes the specimen asymmetrical which eliminate symmetrical problems that arrive when applying the damage assessment framework. The dimensions of the specimen can be seen in Figure 2.

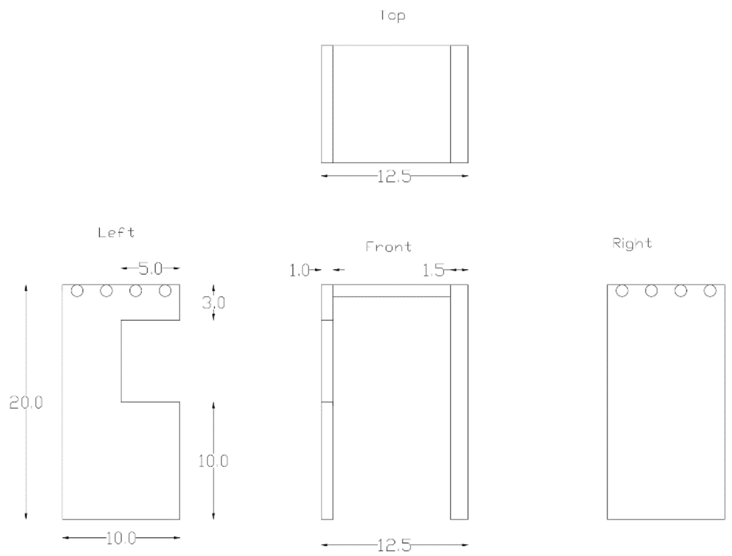


Figure 2: Dimensions of laboratory specimen

The critical failure mechanism of the specimen is the bolts. Therefore, these are the area of interest in upcoming analysis of this paper. The numbering of the eight bolts is showcased in Figure 3.

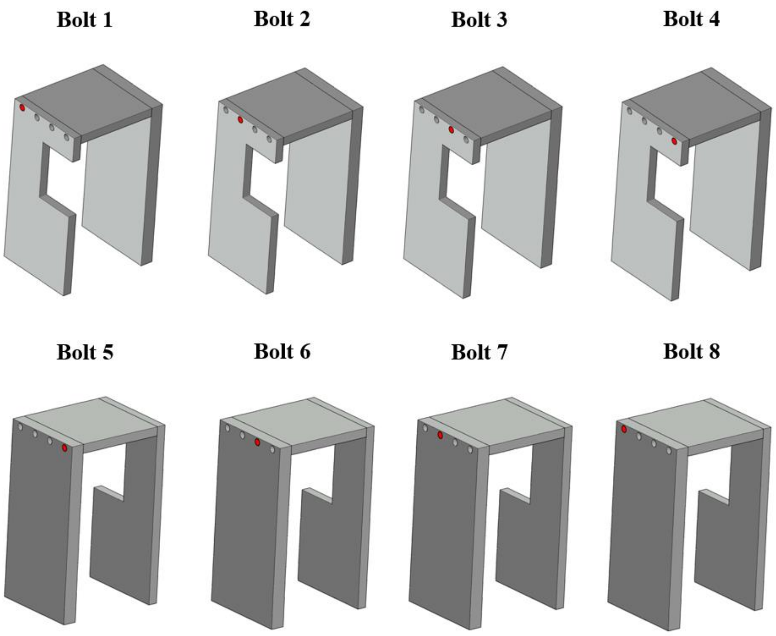
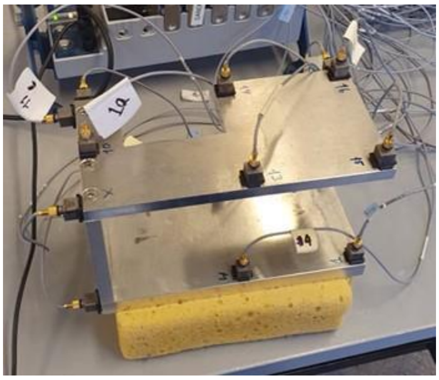
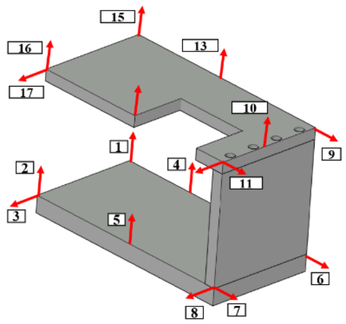


Figure 3: Numbering of bolts

4.1 Experimental Testing

The procedure of the experimental testing will be explained in this section. The testing will be classic Operational Modal Analysis testing, where the specimen will be excited by random vibrations. A total of 17 uni-axial accelerometers are mounted on the U-specimen to record the excitation. The specimen is placed on a sponge to eliminate the effect of boundary conditions and resemble a free-free testing condition. The setup of the specimen is observed in Figure 4a, whereas the position of the accelerometers is depicted in Figure 4b.





(a) Test setup

(b) Placement of accelerometers

Figure 4: Experimental setup

Once the setup is completed, a scratch test is performed where a wooden object is used to scratch the corners of the specimen which initiate random vibrations. The excitation is recorded and converted into the frequency domain to decide the modal parameters. The frequencies are excerpted by using Enhanced Frequency Domain Decomposition (EFDD) where the singular values from the spectral matrix are determined [9]. The plot of the power spectral density function for a reference test in the undamaged state is visualized in Figure 5, where the peaks resemble the frequency for the modes.

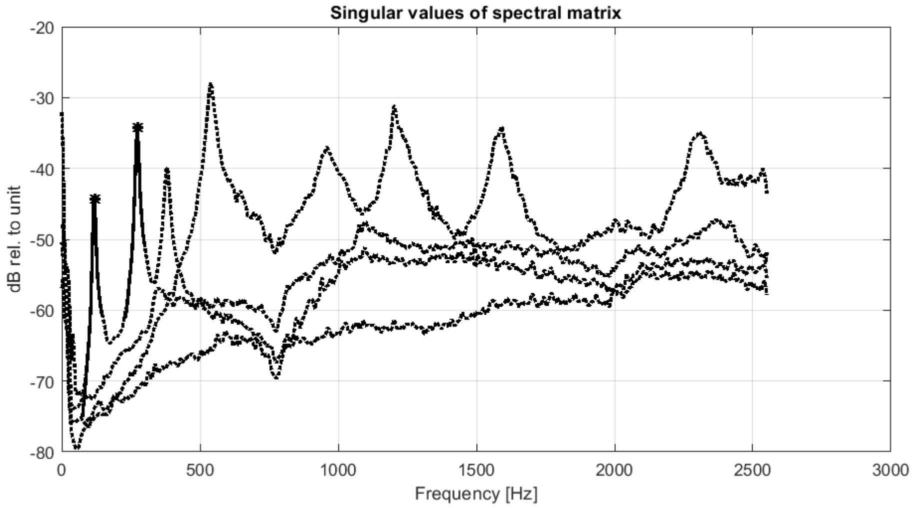


Figure 5: Plot for singular values

The first eight modes can be derived from the Frequency Domain Decomposition analysis. These modeshapes can be seen in Figure 6 along with their corresponding natural frequency.

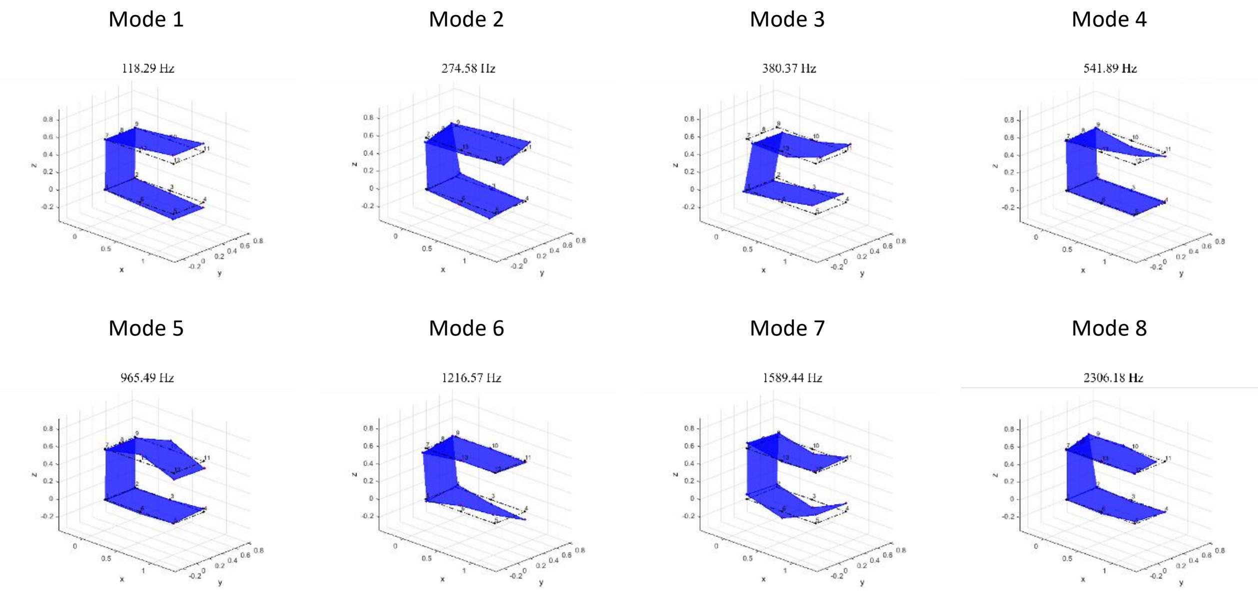


Figure 6: Experimental modeshapes for the first 8 modes

5. FE-Model

A simple parameterization of the laboratory specimen has been performed in the commercial software ANSYS- APDL. The FE-elements SHELL181 are utilized to model the specimen, which are 4-node shell elements with six degrees of freedom at each node. Figure 7 depicts the meshed model with the boundary conditions which are six springs with a longitudinal stiffness of $k = 100 N / m$

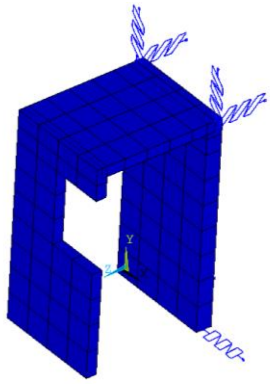


Figure 7: FE-Model

Finally, the properties of the FE-Model are presented in Table 1.

Element Type	No. Elements	Density	E-modulus	Poisson Ratio
[-]	[-]	[kg/m3]	[GPa]	[-]
SHELL181	86	2850	6.7	0.34

Table 1: FE-Model Properties

FE-Model Updating

The accuracy of the presented FE-model is predominately dependent on the choice of finite elements, the mesh, the modeling of joints and matching of the real material properties. Adjusting the mentioned parameters are performed until the FE-Model can be regarded as a satisfactory representation of the experimental model. The FE-model can primarily be validated by comparing the numerical modal parameters with the experimentally obtained ones. These include the natural frequencies and the mass-normalized modeshapes. The natural frequencies are compared by calculating the relative deviation for each mode. As for the modeshapes, the original formulation of the MAC presented is used to determine the linear correlation between a pair of modes. The validation of the frequencies is observed in Table 2 for the first eight pairs. It is observed that numerical frequencies deviate 4.68% - 11.30% for most of the modes. Even though these deviations are considerably large they are still acceptable. The MAC value for the first 9 modes is displayed in Figure 8.

Pair	FEM-Frequency	OMA-Frequency	Deviation	MAC
[-]	[Hz]	[Hz]	[%]	[%]
1	112.99	118.28	4.68	99.27
2	271.19	274.58	1.25	99.39
3	354.33	380.37	7.35	98.30
4	591.99	541.89	-8.46	97.55
5	994.97	965.49	-2.96	97.05
6	1211.50	1216.57	0.42	96.98
7	1597.10	1589.44	-0.48	95.67
8	2072.00	2306.18	11.30	90.25

Tabel 2: Mode pairs between experimental tests and FE-Model

Apparent from Figure 8 is that there is a satisfactory correlation between the modeshapes for the first 8 modes, which permanently disregard mode 9 for further analysis. The final values of the MAC are included in Figure 8.

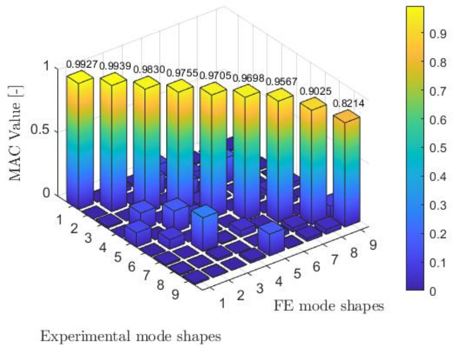


Figure 8: MAC validation for modeshapes

5.1 Definition of Predefined Damage Scenarios

Once the FE-model is presumed to be an adequate description of the laboratory specimen, the sensitivity matrix $\mathbf{S}$ must be constructed by producing several predefined damage scenarios. In this case study, eight predefined damage locations will be parameterized where each scenario will represent a damage in a bolt as showcased in Figure 9.

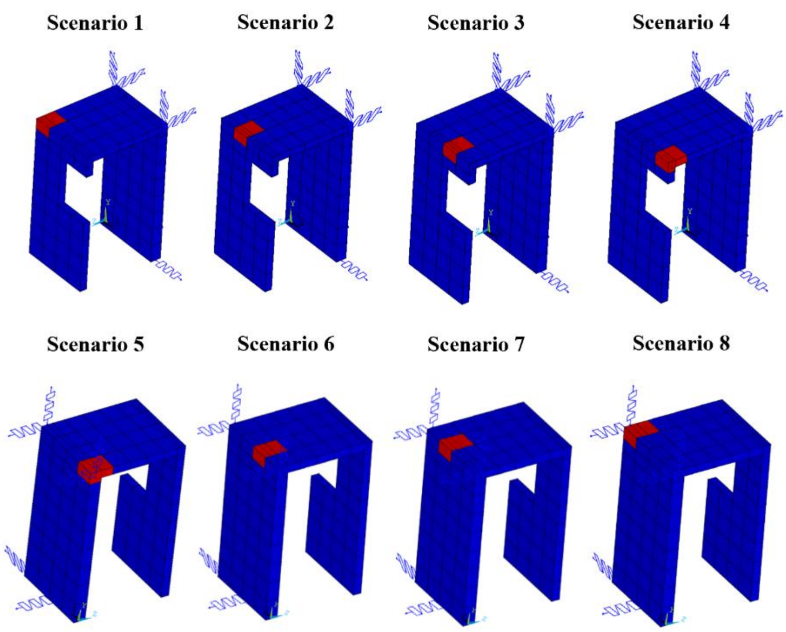


Figure 9: Scenario Definitions

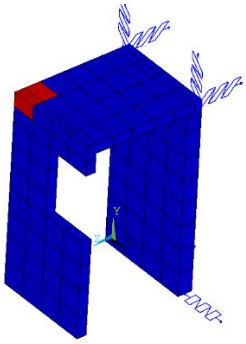
It is observed that for each scenario, two adjacent shell elements have predefined damage where the Young-Modulus is reduced. Moreover, three sensitivity matrices will be constructed based on the reduction of the Young-Modulus, $\mathbf{S}_{15}$, $\mathbf{S}_{50}$ and $\mathbf{S}_{75}$, which designate 15%, 50% and 75% E-module reductions.

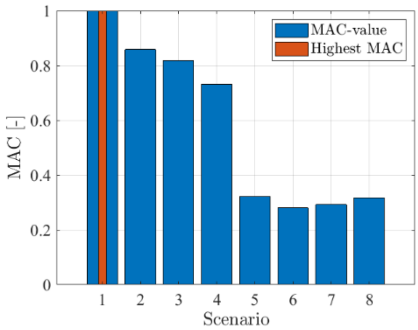
5.2 Numerical Simulation

To better understand the implementation of the proposed damage assessment procedure, a numerical simulation of the specimen is studied in this section to evaluate the robustness of the Level 2 and Level 3 approaches. The numerical simulation will generate damage examples in the same way as the predefined scenarios are formulated, namely by the reduction of the Youngs Modulus.

Localization through MAC-SBDA

The MAC-SBDA approach acts solely as a damage localizer, hence it is crucial to inspect the accuracy of the approach by simulating damages in various locations on the FE-model. For each unique simulation, the change of frequency will be stored in the vector $\Delta\mathbf{f}$. As a starting point, a damage is simulated in a location identical to the first predefined scenario.

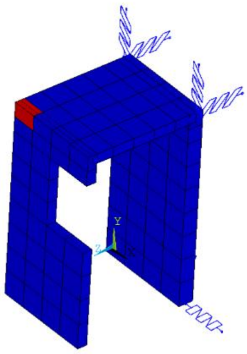


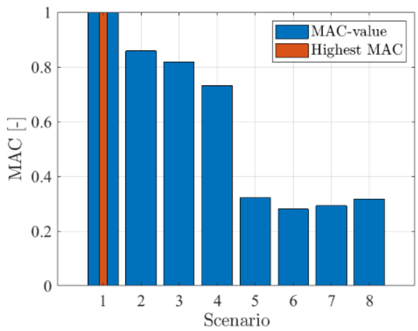


(a) Damage in location 1 (b) MAC-distribution

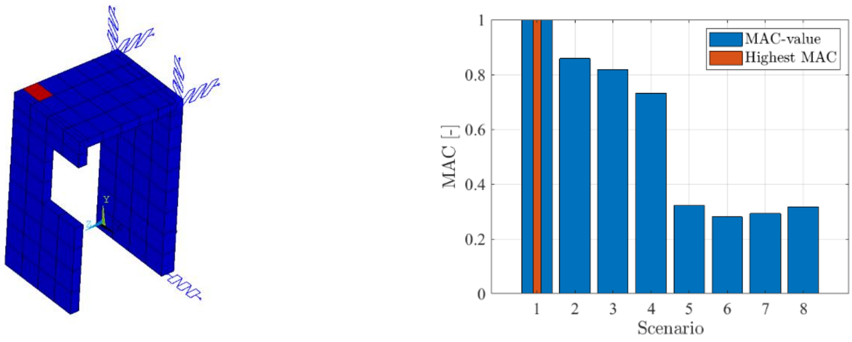
Figure 10: MAC-SBDA on damage location 1

Figure 10b showcases the results, and it is verified that the damage is indeed localized correctly, which is expected since the simulated damage is fundamentally equal to the first column of sensitivity matrix, in other words - scenario 1. Simulations for damage reduction for a single element in location 1 are carried out in Figure 11. Again, the correct scenario is localized.





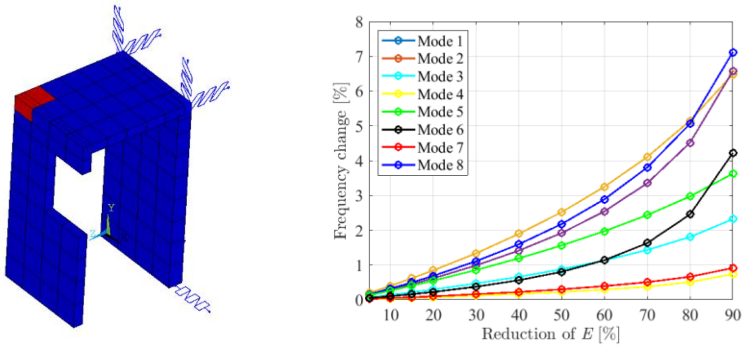
(a) Damage in location 1 (Single element) (b) MAC-distribution



(c) Damage in location 1 (Single element) (d) MAC-distribution

Figure 11: MAC-SBDS on damage location 1 (single elements)

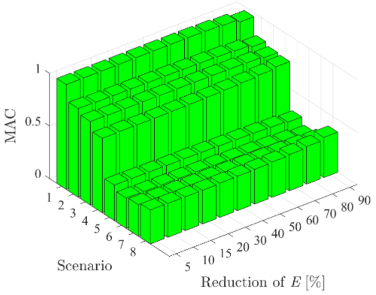
The influence of damage intensity on the MAC-SBDA is now examined through a simulation study where frequency changes for damage cases in the range of 5 % to 90 % are computed. The simulations of damage are carried out on location 1, which is as stated, equivalent to the first predefined scenario. Firstly, the relation between the frequency changes and the E-Modulus reduction are portrayed for the first 8 modes in Figure 12.



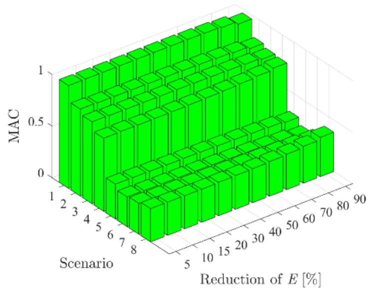
(a) Damage location 1 (b) Frequency changes [%]

Figure 12: Frequency changes for simulation study

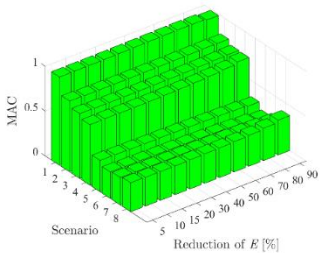
The frequency changes follow predominantly linear behavior for damages up to 60 %. Eventually nonlinearity develops, where some of the eigenmodes begin to interchange. The reasoning behind this phenomenon is the fact that the structure has reached a plastic state where there is a clear change in the eigenmodes. It is therefore necessary to discover if damages in the nonlinear segment can be localized through the MAC. Figure 13 showcase the calculated MAC-SBDA for the performed simulations for the three sensitivity matrices $\mathbf{S}_{15}$, $\mathbf{S}_{50}$ and $\mathbf{S}_{75}$.



(a) MAC distribution using $\mathbf{S}_{15}$



(b) MAC distribution using $\mathbf{S}_{50}$



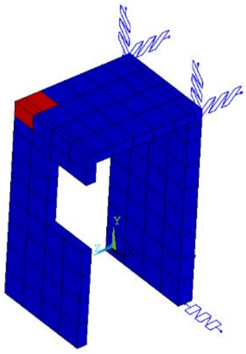
(c) MAC distribution using $\mathbf{S}_{75}$

Figure 13: MAC-SBDA using the three sensitivity matrices

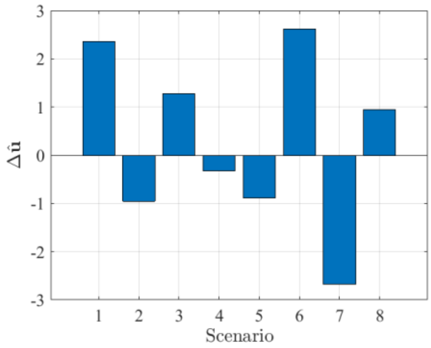
It is observed that the intensity of damage does not affect the accuracy of the location of the damage even at damages where nonlinear behavior is apparent. Theoretically, the only way for the MAC to localize the wrong damage in a numerical analysis is if the predefined scenarios have indifferent frequency changes. This is a general case for symmetrical structures, which results in more modes required.

Quantification Through a Pseudo-inverse Approach

Quantification is the subsequent step to localization in a Rytter's damage identification scheme. Numerically, it must be proved whether the pseudo-inverse approach can estimate the extent of a damage accurately. To demonstrate the concept of quantification through the pseudo-inverse approach, a 30 % damage is simulated in damage location 1 where the frequency changes for the first 8 modes are stored in $\Delta\mathbf{f}$. For the sensitivity matrix, $\mathbf{S}_{15}$ is utilized containing all eight predefined scenarios. Since the number of modes are equal to the number of existing scenarios, the system of equations is determined, and expression (6) is adopted. The calculated $\Delta\mathbf{u}$ is visualized as a bar plot in Figure 14b.



(a) Damage location 1



(b) Damage quantification index

Figure 14: Quantification of damage location 1

Estimating the damage extent in Figure 14b can now be done by reading off the value in the correct localized scenario. Since the damage is in scenario 1, the value of the first bar is read off. This value is estimated to be $\Delta\mathbf{u} = 2.36$. Since the reference reduction of E-modulus is 15% for the chosen sensitivity matrix $\mathbf{S}_{15}$, the estimated value can be multiplied with 15% which yields,

$$\Delta\mathbf{u} = 2.36 \cdot 15\% = 35.4\%$$

It is observed that the estimated damage is remarkably close to actual induced damage of 30% E-modulus reduction.

As mentioned in the theory section, an overdetermined system of equations can yield better results when utilizing the pseudo-inverse method. A similar simulation study to the MAC is conducted, where the Youngs Modulus is reduced between 5-90%. The simulation is performed for sensitivity matrix $\mathbf{S}_{15}$ with three different matrix dimensions. The results are showcased in Figure 15,

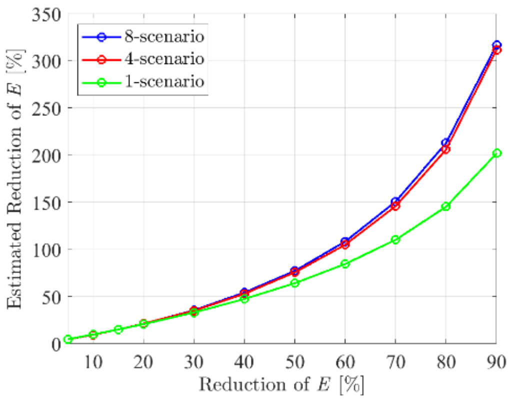


Figure 15 – Influence of number of scenarios

It is detected that by neglecting the other 7 scenarios and only using an $\mathbf{S}$ containing the correct scenario, the accuracy of the damage extent is improved mostly for all damages over 40%. Still, the quantification follows nonlinear behavior where the largest damages have unrealistic estimations. The nonlinear behavior can be explained by the frequency changes which were presented in Figure 12b.

To combat the improbable estimations of the larger damages, a fitting sensitivity matrix can be chosen. As previously asserted, three different sensitivity matrices were defined, each with a different reduction in the Young's Modulus. The impact of these sensitivity matrices on the quantification for damage location 1 is examined in Figure 16. It is observed that the accuracy of the quantification depends heavily on the definition of the sensitivity matrix. Small stiffness reductions are better quantified when using a sensitivity matrix that is defined by small stiffness reductions. On the contrary, larger stiffness reduction are better quantified employing a sensitivity matrix with already high stiffness reductions.

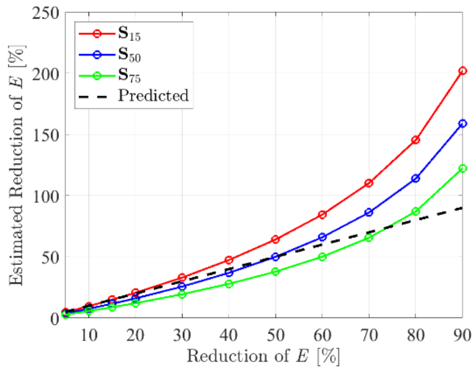


Figure 16: Influnece of sensitivty matrix when simulation damages between 0% and 90% on location 1

5.3 Damage Assessment of Case Study

This section will apply the 3-level damage identification framework on presented case study in an experimental setting, where individual structural perturbations be detected, localized, and quantified. The section will mainly demonstrate the framework on bolt 1 as an exemplification, however observations for other bolts will be mentioned and discussed.

In the undamaged state, all eight bolts have a torque of $M_{undamaged} = 3\ \mathrm{Nm}$. A perturbation will then be defined by reducing the torque using a torque wrench. Three different levels of damage intensities on each bolt will be considered.

- Damage intensity 1: Reduction of torque to 2.0 Nm which equals to a 33% relative loss of stiffness of the individual bolt
- Damage intensity 2: Reduction of torque to 1.5 Nm which equals to a 50% relative loss of stiffness of the individual bolt
- Damage intensity 3: Reduction of torque to 0 Nm which equals to a 100% relative loss of stiffness of the individual bolt

The reason behind the choice of three levels of damage intensities is to investigate at which state damage can be detected. On the other hand, it is also necessary to ascertain if the provided damage identification framework can distinguish between two damages that are close to each other in magnitude, hence the choice of damage intensity 1 and 2

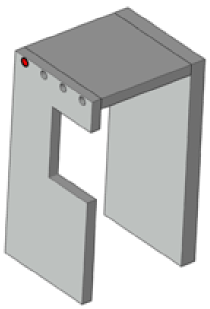
Damage Detection

Before performing the approaches for Level 2 localization and Level 3 quantification, a damage must be detected.

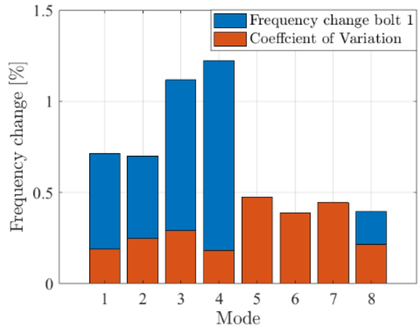
Detection of damage can solely be performed using statistical measures of the monitored specimen. In this paper, a statistical method called the coefficient of variance is utilized. Calculating the coefficient of variance of the frequencies of each mode for a series of reference test provides an idea of the dispersion. It any frequency change is larger than the calculated dispersion, damage can be assumed to be detected.

$$\frac{\omega_{bi}-\omega_{ai}}{\omega_{bi}} > \frac{\sigma_{\omega_{bi}}}{\mu_{\omega_{bi}}} \tag{8}$$

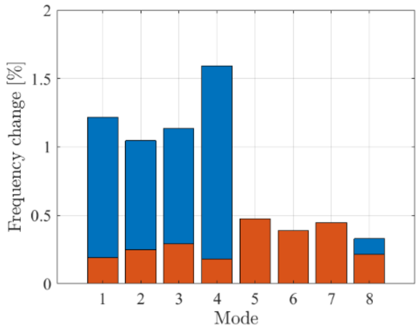
For bolt 1, the coefficient of variance is calculated and plotted with normalized frequency changes for each damage intensity.



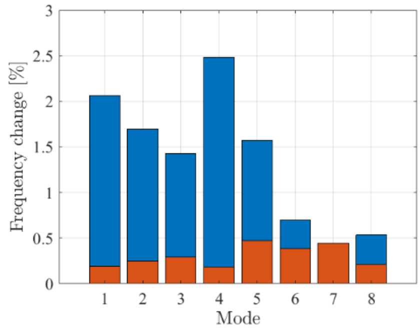
(a) Bolt 1



(b) 2 Nm torque



(c) 1.5 Nm torque



(d) 0 Nm torque

Figure 17: Frequency changes for damage for bolt 1 along with the coeffcient of variance

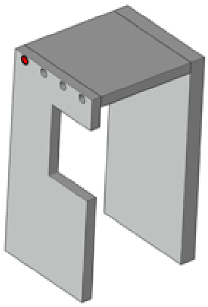
Apparent from Figure 17, damage is clearly detected since the frequency changes are far greater than the dispersion. In addition, the frequency changes grow larger as the damage intensities increase which is of course anticipated.

Damage Localization

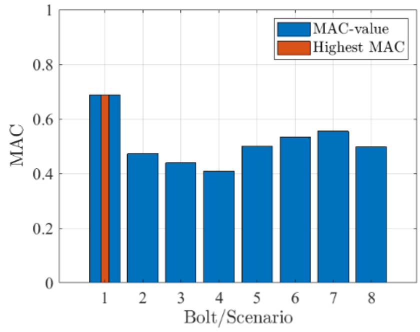
The MAC-SBDA can now be utilized to compare the measured frequency patterns from the experiments, with the hypothetical damage scenarios stored in the sensitivity matrix. Examples will be provided for Bolt 1, Bolt 4, and Bolt 5.

Bolt 1

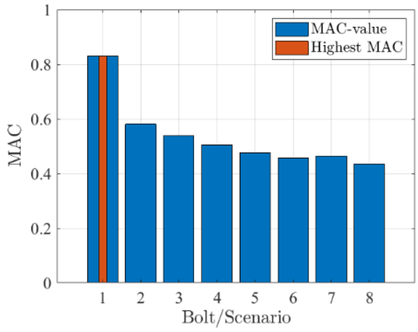
Figure 18 display the MAC distribution for the three damage intensities concerning bolt 1. It is observed that the correct scenario is indeed localized in all instances with varying MAC-values. More explicitly, the damage case with best correlated MAC pattern, in accordance with parametric scenarios, is when the bolt has 0 torque.



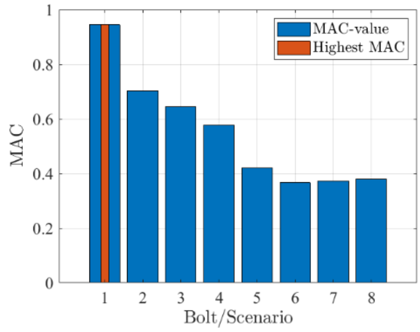
(a) Bolt 1



(b) 2 Nm torque



(c) 1.5 Nm torque

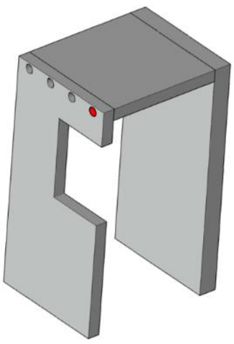


(d) 0 Nm torque

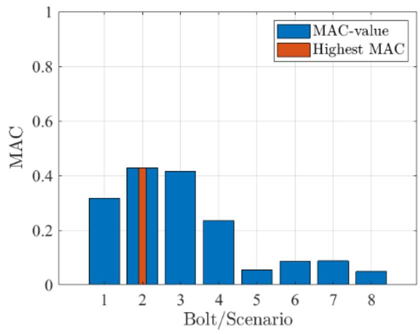
Figure 18: MAC-SBDA for bolt 1

Bolt 4

The MAC-SBDA is likewise performed for OMA-tests on bolt 4. Here it observed that the correct damage cannot be localized. Damage in this bolt is insufficient to the global behavior for the dynamic properties since it does not impact the stiffness of the joint. Even having a torque of 0 Nm, showcase very little frequency changes which are less than the noise floor. Therefore $\Delta\mathbf{f}$ will just be a containment of the noise and the correct scenario cannot be localized as seen in Figure 19.



(a) Bolt 4

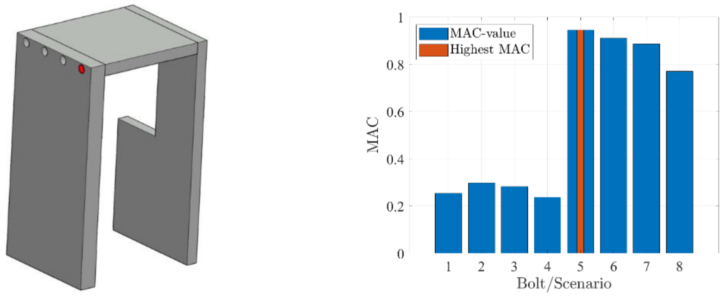


(b) 0 Nm

Figure 19: MAC distribution for damage on bolt 4

Bolt 5

A last example to showcase the MAC's robustness experimentally is a damage on bolt 5. Here the torque is reduced to 0 Nm. Again, the correct damage is localized.



(a) Bolt 5 (b) 0 Nm

Figure 20: MAC-distribution for damage on bolt 5

Damage quantification

The final step in the 3-level damage identification framework is to quantify the structural perturbation by using the pseudo-inverse approach. Here, it is important to highlight that the experimental damage varies from the parametric scenario definitions in terms of physical units. The hypothetical damage in the parametric model is defined as the reduction of Young's-Modulus whereas the experimental damage is governed by torque reductions. However, the relative reductions in percentage can still be compared since proportionality exists in the pseudo-inverse approach. Additionally, only damages that are localized correctly can be quantified, since the correct scenario is chosen before solving the system of equations in the pseudo-inverse approach. Therefore, estimating the damage extent for perturbations for bolt 4 are not possible. Finally, it must be noted that the discoveries discussed in the numerical analysis will all be implemented. Only the columns containing the correct localized scenario in the $\mathbf{S}$ will be utilized as the sensitivity matrix. In addition, the best fitted sensitivity matrix will be used to quantify the damage.

For bolt 1, the following quantifying value are estimated.

Torque	2 Nm	1.5 Nm	0 Nm
Relative stiffness loss	33%	50%	100%
Quantified value	**18.64%**	**26.8%**	**48.33%**

It is seemingly seen that the pseudo-inverse approach yields realistic results for the damage extent, however it still deviates from the relative stiffness loss.

It must be noted that it is improbable that accurate estimations of the damage extent are achieved. The MAC proved that even though the correct scenario is localized, the general MAC values for the bolt is not very high. This indicates that the correlation between the measured frequency changes and the parametrized scenarios are not the same. Therefore, it cannot be expected that increments for damages in the parametrized scenarios are synonymous with increments in damage for the real-life damage cases.

6. Conclusion

The application of vibrations based Structural Health Monitoring and frequency-based damage assessment has been demonstrated on a bolted laboratory specimen, by performing a 3-level damage identification scheme that is essentially established on a scenario-based framework. Both numerical and experimental investigations were executed to validate the identification scheme.

The numerical study scoped out Level 2 and Level 3, where the MAC-SBDA approach and the pseudo-inverse approach were applied. The MAC-SBDA showcased promising results numerically. It was possible to localize different locations of stiffness reductions in FE-model, even in the nonlinear segment. The degree of damage only affected the localization for smaller frequency changes where noise was apparent

Quantification of the damages through a pseudo-inverse approach in the numerical model proved to be robust if the best fitting sensitivity matrix was chosen and only using the column containing the correct localized scenario. It was evident that nonlinear behavior was present when quantifying different levels of damage, where larger stiffness reductions yielded unrealistic results. However, with the correct sensitivity matrix, even larger damages in nonlinear segment could be quantified.

Experimentally, the 3-level damage identification scheme was applied where the torque on each individual bolt was reduced. Three damage intensities were covered, and the goal was to discover at which intensity damage could be detected. Identifying the presence of damage was done by using the statistical approach called the coefficient of variance. It was proved that damage could be detected for all three damage intensities.

Moving to Level 2, localization was performed by using the MAC-SBDA. The MAC-SBDA method highlighted the correct damage locations in almost all instances expect for position where the stiffness of the bolt is insignificant to the global dynamic properties.

Lastly, the experimental damages were quantified solving the system of equations in the pseudo-inverse approach. Only the correct localized scenarios were quantified, and it was observed that realistic values for the quantification was recorded.

Overall, the 3-level damage identification scheme proved to be a robust framework when applying it on the laboratory specimen.

7. Bibliography

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