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Point foundation. Upper and lower bound solution for point load and upper bound solution for distributed load

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Abstract

First, the case where the concentrated load, P, acts as a point on the upper face of the foundation is considered. Facing the bottom, an evenly distributed reaction acts, p. Subsequently, the case where the concentrated load acts over a finite area is considered.

The capacity will first be determined for a circular foundation subjected to a point load. This capacity is achieved by using both the upper and lower bound theorems shown in section 1 and 2 respectively. It is shown here that the solution is exact. The upper bound solution for a foundation affected by a point load at one point is well known, but is included here to show similarities and differences compared to the foundation affected by a point load acting over a finite area.

Subsequently, a square point foundation affected by a point load acting at a point is examined in section 3. Here it can be seen that the capacity of a circular and a square point foundation has the same capacity as long as the area of the two is equal.

In section 4, an upper bound solution is drawn up for a circular foundation affected by a point load acting over a finite area.

1 Upper bound solution for foundation affected by concentrated load acting at a point at the centre

Figure 1 shows a local mechanism under a point foundation for a concentrated load, P, acting at a centrally located point. The load is lowered a distance δ, and the breaking mechanism forms a cone. The angular rotation between the part of the foundation that does not move in connection with the formation of the mechanism and the inclined sides of the mechanism is $\frac{\delta}{R}$, where R is the radius of the mechanism in the plan. In the following, it is assumed that the negative moment capacity is 0, m_p` = 0. It is also required that the size of the foundation is larger than what is required to be able to form the optimal mechanism. The yield lines running from the center of the cone are positive and the moment capacity is m_p.

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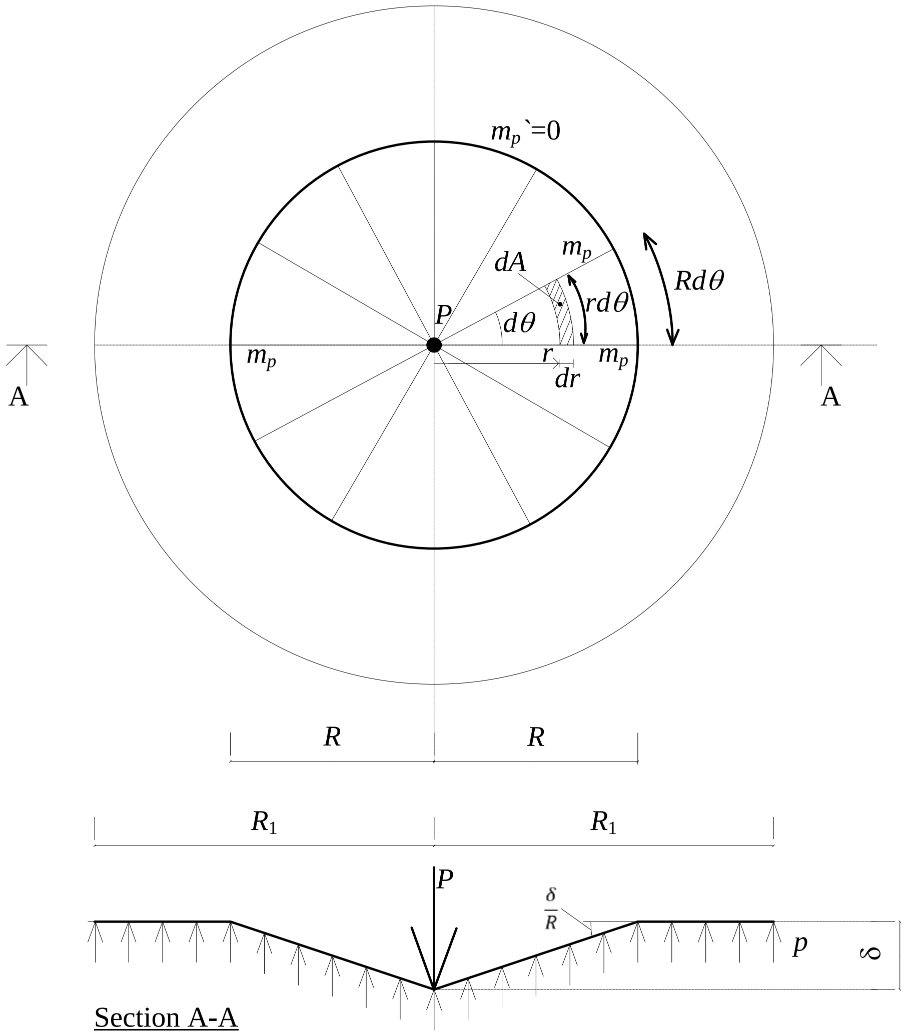


Figure 1 Mechanism under point load

1.1 External work

The external work carried out by the concentrated load will be positive as the concentrated load and its displacement are in the same direction. Conversely, the work carried out by the distributed reaction will be negative since the reaction and its displacement are directed opposite. The reaction comes from the point load alone, since the positive contribution of the dead load is equalized by the reaction from the dead load.

$$A_y = P \cdot \delta - \int_0^R \int_0^{2\pi} \frac{R-r}{R} \delta \cdot p \cdot r d\theta \cdot dr \qquad (1)$$

$$A_y = P \cdot \delta - \frac{1}{3}\pi \cdot p \cdot R^2 \cdot \delta \qquad (2)$$

1.2 Internal work

The internal work is generally given by:

$$A_i = \int_0^R \int_0^{2\pi} m_p \cdot dv \cdot dr \qquad (3)$$

where *dv* is the angular rotation perpendicular to the yield moment m_p.

The angular rotation perpendicular to the moment vector is determined based on figure 2. The normal to the radial lines, which are located at the same distance from the center, meet at the same point placed above the center. Figure 2 shows the normals belonging to any two radial lines shown in figure 1 positioned with an angle $d\theta$ between them. The normals are located at a distance R from the center. The lengths of the normals are denoted b.

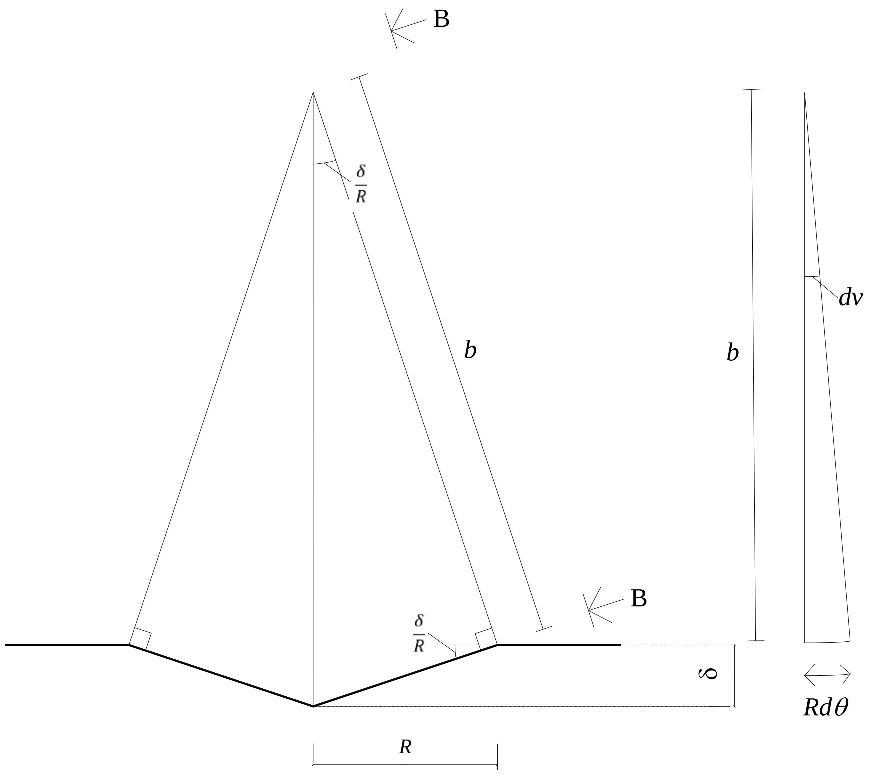


Figure 2 Section A-A from figure 1, sketched without point load and reaction

By inserting a section B-B as shown, it can be seen that the angular rotation between the two normals is dv. On section A-A it can be seen that the angle between the normals and the vertical is $\frac{\delta}{R}$. The length of b is given by

$$tan\frac{\delta}{R} = \frac{R}{b}$$
$$\frac{\delta}{R} \sim \frac{R}{b}$$
$$b = \frac{R^2}{\delta}$$

As we are talking about infinitesimal angles, we allow ourselves to approximate.

The angle dv can then be expressed as:

$$dv \cdot b = Rd\theta$$
$$dv = \frac{R}{b} d\theta$$

With the expression for b inserted

$$dv = \frac{\delta}{R} d\theta$$

Inserted in the expression for the internal work:

$$A_i = \int_0^R \int_0^{2\pi} m_p \cdot \frac{\delta}{R} d\theta \cdot dr \qquad (4)$$
$$A_i = 2\pi \cdot m_p \cdot \delta \qquad (5)$$

From the work equation:

$A_y = A_i$

$P \cdot \delta - \frac{1}{3}\pi \cdot p \cdot R^2 \cdot \delta = 2\pi \cdot m_p \cdot \delta$

$$m_p = \frac{P}{2\pi} - \frac{1}{6}p \cdot R^2 \qquad (6)$$

The maximum value is immediately seen to be obtained by letting the free parameter R go towards 0; i.e. a cone with very steep sides. Thereby the following is obtained:

$$m_p = \frac{P}{2\pi} \qquad (7)$$

This solution is also found in [1].

2 Lower bound solution for foundation affected by concentrated load acting at a point at the centre

Consider the case where the concentrated load, P, acts as at point in the center once again. The reaction is again calculated evenly distributed with the intensity $p = \frac{P}{\pi R^2}$.

In accordance with the upper bound solution, it could be obvious to calculate the moment along the radial lines constantly with the value m_θ.

The magnitude of this constant moment can be determined by taking moment equilibrium about a line through the center. The result of this gives:

$$m_\theta = \frac{P}{3\pi} \qquad (8)$$

By utilizing the boundary conditions $v_r(r = R) = m_r(r = R) = 0$ and $v_\theta = m_{r\theta} = 0$ from the rotational symmetry, it is obtained from the equilibrium equations of the plate in polar coordinates that:

$$m_r = -\frac{P}{6\pi} \cdot \left(1 - \left(\frac{r}{R}\right)^2\right) \qquad (9)$$

Assuming that m_θ is constant thus leads to a need for reinforcement in both top and bottom. The sum of the reinforcement in top and bottom is identical to the need found in the upper bound solution. This solution can be found in [2]. Since the upper bound solution only requires reinforcement in the bottom, while the lower bound solution requires reinforcement in both bottom and top, the solution is not the same and therefore not exact.

As seen, assuming a constant value of m_θ results in a negative value of m_r. An illustration of this can be seen by looking at a section at the edge shown in figure 3. The constant value of m_θ on the outermost part is greater than what is needed to create equilibrium of this section, therefore m_r must be directed opposite, and thus negative.

The obvious next suggestion could be to assume $m_r = 0$ throughout the plate. Figure 3 shows a horizontal section in the foundation with radius R. With thick the adjacent line of the foundation's perimeter is shown along with two radial lines that form the angle θ with each other and a circular section at a distance r from the centre.

The figures are strongly plotted with respect to the magnitude of $d\theta$. $d\theta$ must be seen as an infinitesimal quantity ($d\theta \rightarrow 0$), which means that $\sin(d\theta) \approx d\theta$.

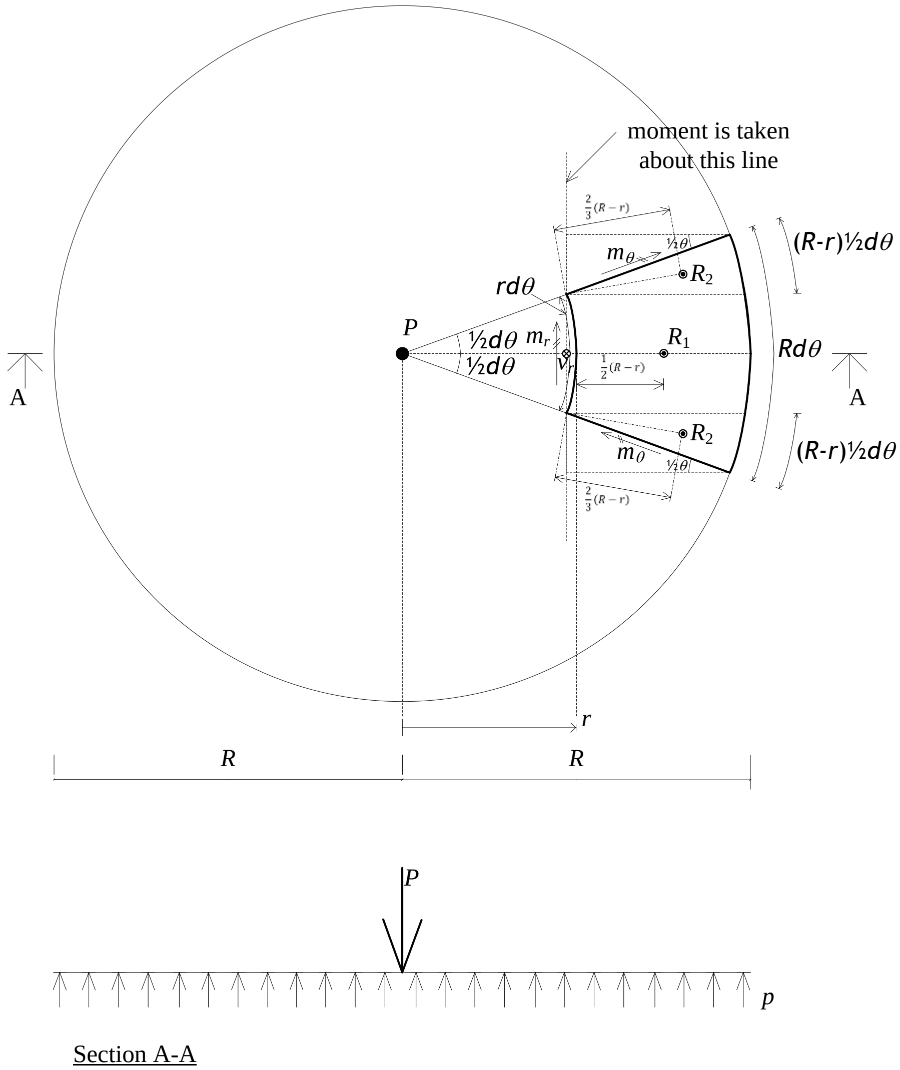


Figure 3

It is noted that it was previously *chosen* that m_θ should be constant. The magnitude of m_θ was found by moment equilibrium of a section through the center to $m_\theta = \frac{1}{3} \cdot p \cdot R^2 \left(= \frac{P}{3\pi}\right)$.

As it is seen below, m_θ is not necessarily constant. In order to achieve moment equilbrium in the section shown in figure 3, the mean value of m_θ going from the center to the periphery is required to take on this value, i.e.:

$$\overline{m_\theta} = \frac{1}{R}\int_0^R m_\theta(r)dr = \frac{1}{3} \cdot p \cdot R^2 \tag{10}$$

Introducing m_r(r) = 0 means that $\frac{\partial(m_r \cdot r)}{\partial r}$ = 0.

The equations of equilibrium in polar coordinates are given by:

1. $$v_r \cdot r = \frac{\partial(m_r \cdot r)}{\partial r} - \frac{\partial m_{r\theta}}{\partial \theta} - m_\theta \tag{11}$$
2. $$v_\theta = \frac{1}{r}\frac{\partial m_\theta}{\partial \theta} - \frac{\partial m_{r\theta}}{\partial r} - 2\frac{m_{r\theta}}{r} \tag{12}$$
3. $$\frac{\partial(v_r \cdot r)}{\partial r} + \frac{\partial v_\theta}{\partial \theta} = -p \cdot r \tag{13}$$
4. $$\frac{1}{r}\frac{\partial^2(m_r \cdot r)}{\partial r^2} - \frac{2}{r^2}\frac{\partial^2(m_{r\theta} \cdot r)}{\partial r \partial \theta} + \frac{1}{r^2}\frac{\partial^2 m_\theta}{\partial \theta^2} - \frac{1}{r}\frac{\partial m_\theta}{\partial r} = -p \tag{14}$$

The starting point is the first equilibrium equation:

$$v_r \cdot r = \frac{\partial(m_r \cdot r)}{\partial r} - \frac{\partial m_{r\theta}}{\partial \theta} - m_\theta \tag{15}$$

from the vertical equilibrium the following is obtained:

$$v_r \cdot r = -\left(\frac{1}{2} p \cdot R^2 - \frac{1}{2} p \cdot r^2\right) \tag{16}$$

As it has been chosen to apply

$$\frac{\partial(m_r \cdot r)}{\partial r} = 0 \tag{17}$$

just as the other boundary conditions still apply, we also have that $\frac{\partial m_{r\theta}}{\partial \theta} = 0$. Therefor the equilibrium equation gives that:

$$m_\theta = \frac{1}{2} p \cdot R^2 - \frac{1}{2} p \cdot r^2 \tag{18}$$

$$m_\theta = \frac{P}{2\pi}\left(1 - \left(\frac{r}{R}\right)^2\right) \tag{19}$$

To check the moment equilibrium about the line through the center, we have

$$\overline{m_\theta} = \frac{1}{R}\int_0^R m_\theta(r) dr \tag{20}$$

Which gives us:

$$\overline{m_\theta} = \frac{1}{3} \cdot p \cdot R^2 \tag{21}$$

This is as required.

The moment distribution is thus given by:

$$m_r(r) = 0 \tag{22}$$

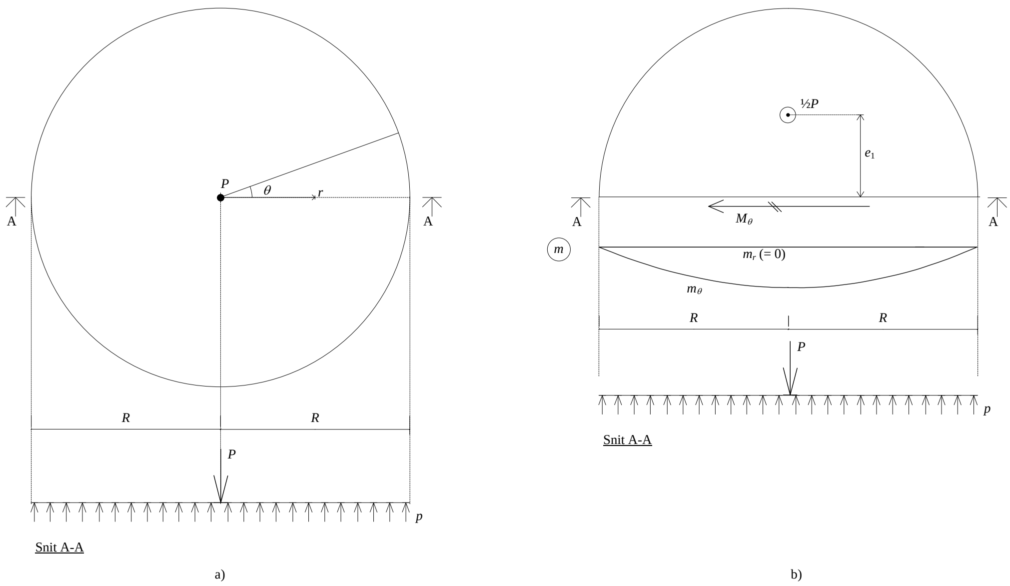
$$m_\theta = \frac{P}{2\pi}\left(1 - \left(\frac{r}{R}\right)^2\right) \tag{23}$$

The moment distribution across the foundation is shown in figure 4.

The maximum value is found for $r = 0$, and is given by:

$$m_\theta = \frac{P}{2\pi} \tag{24}$$

The result is seen to be identical to the upper bound solution, and like the upper bound solution only requires reinforcement at the bottom of the foundation. The solution is thus exact. Where the moment distribution for the upper bound solution is not known outside the yield lines, the opposite applies for the lower bound solution. The upper bound solution thus requires full capacity throughout the plate and thereby also at the edge. As a consequence of this, the reinforcement must be anchored at the edge (typically achieved by bending up the reinforcement at the edge). Since the moment distribution is known throughout the foundation by the lower bound solution, including that $m_r(r) = 0$ and m_θ varies parabolically with a maximum value in the centre and 0 at the edge, anchoring of reinforcement can often be left out, but this must be assessed in the specific case.



Figur 4 a) static model of circular foundation affected by point load. b) moment distribution

In Appendix 1, a check of the four equilibrium equations is carried out.

3. Lower bound solution for a square slab/foundation subjected to a centrally located concentrated load and supported by a uniformly distributed reaction

We look at a section of a square foundation in Figure 5. This section is shaded and bounded by
$0 \leq x \leq ½l$
$0 \leq y \leq x$

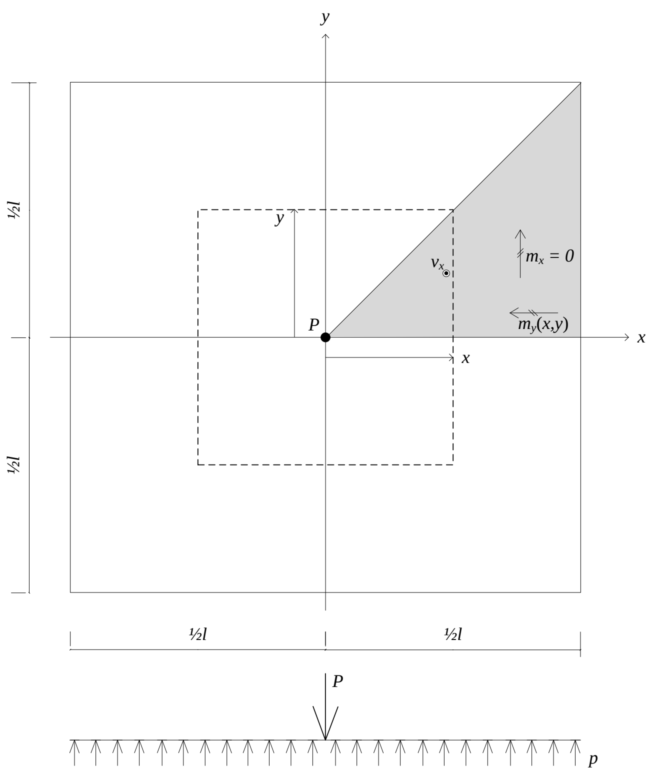


Figure 5 square foundation

The other seven corresponding sections will have a similar moment distribution.

Inspired by the corresponding study of the circular plate, it will be obvious to try with

$$m_x = 0 \tag{25}$$

since there is an obvious similarity between m_r and m_x along the radial-/horizontal axis.

Then an expression for v_x is set up. v_x is considered to be conducted evenly distributed through a section shown with a dashed line in the distance x:

$$v_x \cdot 8x = -(l^2 - (2x)^2)p \tag{26}$$

$$v_x = -\frac{1}{8}pl^2\left(\frac{1}{x} - \frac{4x}{l^2}\right) \tag{27}$$

The following equation of equilibrium applies

$$v_x = \frac{\partial m_x}{\partial x} + \frac{\partial m_{xy}}{\partial y} \tag{28}$$

Since m_x is constantly equal to zero in the shaded area, we have:

$$m_{xy} = \int v_x \partial y \tag{29}$$

$$m_{xy} = -\frac{1}{8}pl^2 \cdot \left(\frac{y}{x} - \frac{4xy}{l^2}\right) + c_1 \tag{30}$$

The constant c_1 is determined by requiring that $m_{xy} = 0$ for $y = 0$, i.e. $c_1 = 0$:

$$m_{xy} = -\frac{1}{8}pl^2 \cdot \left(\frac{y}{x} - \frac{4xy}{l^2}\right) \tag{31}$$

This is seen to give zero (constant) at the edge ($x = ½l$) and thus neither a distributed reaction nor corner forces will occur. This is as required.

m_y is determined by the equilibrium equation of the plate:

$$\frac{d^2m_x}{dx^2} + 2\frac{d^2m_{xy}}{dxdy} + \frac{d^2m_y}{dy^2} = p \tag{32}$$

where

$$\frac{d^2m_{xy}}{dxdy} = -\frac{1}{8}pl^2 \cdot \left(-\frac{1}{x^2} - \frac{4}{l^2}\right) \tag{33}$$

Leading to:

$$\frac{\partial m_y}{\partial y} = \int \left(-2 \cdot \frac{1}{8}pl^2 \cdot \left(\frac{1}{x^2} + \frac{4}{l^2}\right) + p\right)\partial y + c_2 \tag{34}$$

$$\frac{\partial m_y}{\partial y} = -\frac{1}{4}pl^2 \cdot \left(\frac{1}{x^2} + \frac{4}{l^2}\right)y + py + c_2 \tag{35}$$

$$m_y = -\frac{1}{8}pl^2 \cdot \left(\frac{1}{x^2} + \frac{4}{l^2}\right)y^2 + \frac{1}{2}py^2 + c_2y + c_3 \tag{36}$$

As boundary condition we have:

$$\frac{\partial m_y}{\partial y}(y = 0) = 0 \tag{37}$$

Which results in:

$$c_2 = 0 \tag{38}$$

On the inclined boundary towards the region above at $x = y$ must apply that $m_y = 0$, since m_y is constantly equal to zero in this region (in the same way as m_x is constantly equal to zero in the shaded region), that is:

$$m_y(x = y) = -\frac{1}{8}pl^2 \cdot \left(\frac{y^2}{y^2} + \frac{4y^2}{l^2}\right) + \frac{1}{2}py^2 + c_3 = 0 \tag{39}$$

$$c_3 = \frac{1}{8}pl^2 \tag{40}$$

Inserted:

$$m_y = \frac{1}{8}pl^2 \cdot \left(1 - \frac{y^2}{x^2}\right) \tag{41}$$

With respect to moment equilibrium of a section parallel to the *x*-axis through the center of the plate ($y = 0$) the following must apply:

$$m_o \cdot l = \frac{1}{2}\frac{1}{2} l \cdot p \frac{1}{2} l \cdot l \tag{42}$$

$$m_o = \frac{1}{8} p l^2 \tag{43}$$

Expressed by P:

$$P = p \cdot l^2 \tag{44}$$

$$m_o = \frac{1}{8} P \tag{45}$$

This is numerically the maximum moment that occurs (corresponding to the largest principal moment).

4. Upper bound solution for concentrated load acting over a finite area

To take into account the physical extent of the column which affects the foundation, the possibility to have a flat piece under the concentrated load is introduced. This is shown in figure 6, where the flat piece has an extent corresponding to a circular area with radius R_0. By introducing this flat piece, a greater contribution to the internal work is obtained, as the angular rotation of the inclined pieces is increased for $R_0 > 0$.

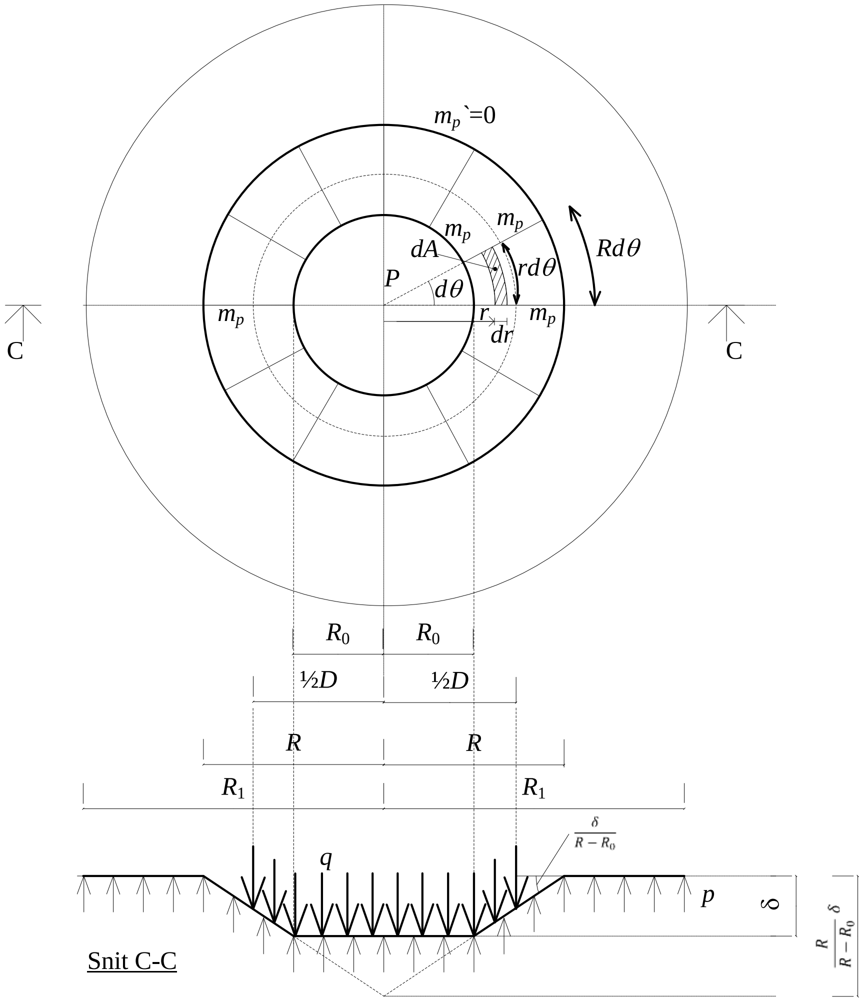


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Figure 6. Local mechanism with the extent of the flat piece at the concentrated load as a smaller than the extent of the concentrated area

External work:

$$A_y = q \cdot \pi(½D)^2 \cdot \delta - \int_{R_0}^{½D} \int_0^{2\pi} \frac{r-R_0}{R-R_0} \delta \cdot q \cdot r d\theta \cdot dr - p \cdot \pi {R_0}^2 \cdot \delta - \int_{R_0}^{R} \int_0^{2\pi} \frac{R-r}{R-R_0} \delta \cdot p \cdot r d\theta \cdot dr \qquad (50)$$

Passer det stadig efter første led er korrigeret til ”½D”

$$A_y = \frac{1}{3} P\delta \left(\frac{1}{D^2} \cdot \frac{1}{R-R_0} \left(3R \cdot D^2 - D^3 - 4{R_0}^3\right) - 4 \cdot \frac{1}{{R_1}^2} \cdot \left(R^2 + R \cdot R_0 + {R_0}^2\right) \right) \qquad (51)$$

Analogous to the review of foundations affected by a single force, the internal work is given by:

$$A_i = 2\pi \cdot \frac{R}{R-R_0} \cdot m_p \cdot \delta \qquad (52)$$

As this is corrected with $\frac{R}{R-R_0}$ to take the larger angular rotation for $R_0 > 0$ into account.

The solution is again determined by:

$$A_y = A_i \qquad (53)$$

$$\frac{1}{3} P\delta \left(\frac{1}{D^2} \cdot \frac{1}{R-R_0} \left(3R \cdot D^2 - D^3 - 4{R_0}^3\right) - 4 \cdot \frac{1}{{R_1}^2} \cdot \left(R^2 + R \cdot R_0 + {R_0}^2\right) \right) = 2\pi \cdot \frac{R}{R-R_0} \cdot m_p \cdot \delta \qquad (54)$$

m_p is isolated:

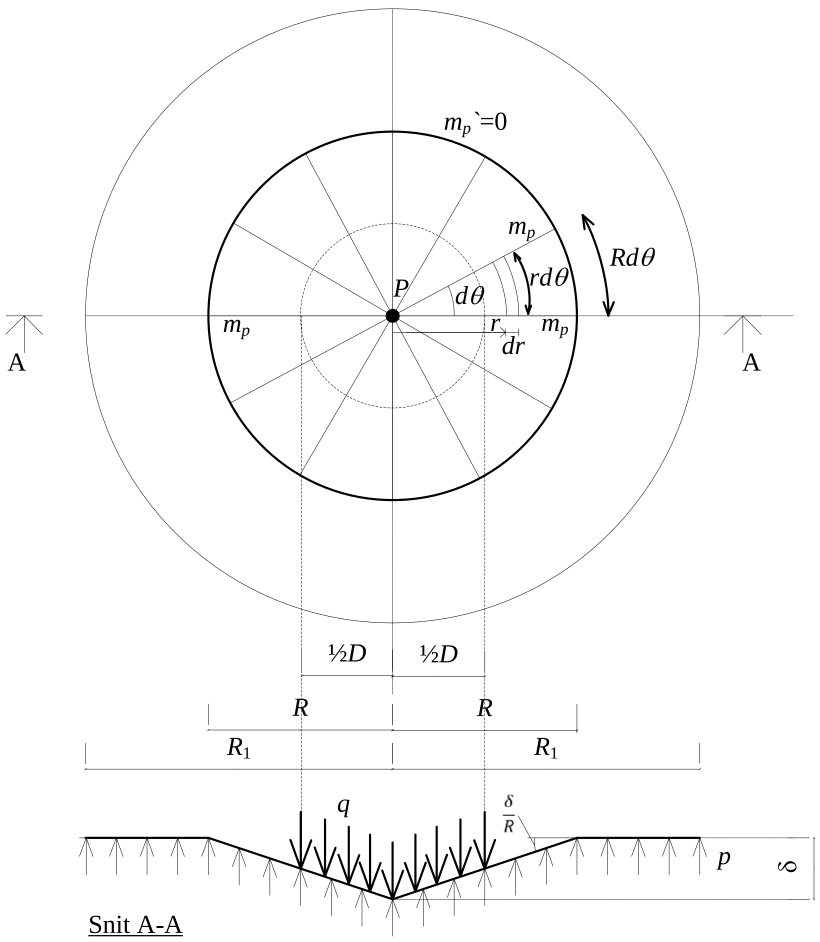
$$m_p = \frac{P}{2\pi} \left(\frac{1}{3} \cdot \frac{1}{RD^2} \left(3R \cdot D^2 - D^3 - 4{R_0}^3\right) - \frac{1}{3} \cdot \frac{1}{R\,{R_1}^2} \left(R^3 - {R_0}^3\right) \right) \qquad (55)$$

The expression which will be optimized with respect to R og R_0:

$$\frac{m_p}{\frac{P}{2\pi}} = \frac{1}{3} \cdot \frac{1}{RD^2} \left(3R \cdot D^2 - D^3 - 4{R_0}^3\right) - \frac{1}{3} \frac{1}{R\,{R_1}^2} \left(R^3 - {R_0}^3\right) \qquad (56)$$

Numerical optimization shows that this mechanism is most critical for $R_0 = 0$. With this, the internal work for the mechanism becomes identical to the internal work in the case of the load acting at a point.

The mechanism corresponding to this is shown in figure 7.



Figur 7 Most critical mechanism

R_0 = 0 inserted in the expression gives:

$$\frac{m_p}{\frac{P}{2\pi}} = \frac{1}{3} \cdot \left(3 - \frac{D}{R}\right) - \frac{1}{3}\frac{R^2}{{R_1}^2} \tag{57}$$

Optimizing with respect to R:

$$\frac{d\left(\frac{m_p}{\frac{P}{2\pi}}\right)}{dR} = 0$$

$$\frac{d\left(\frac{m_p}{\frac{P}{2\pi}}\right)}{dR} = \frac{1}{3} \cdot \left(0 + \frac{D}{R^2}\right) - \frac{2}{3} \cdot \frac{R}{{R_1}^2}$$

The solution to this equation is:

$$R = \sqrt[3]{\frac{D{R_1}^2}{2}} \tag{58}$$

Inserted into the equation for the solution is obtained:

$$\frac{m_p}{\frac{P}{2\pi}} = 1 - \sqrt[3]{\frac{D^2}{4{R_1}^2}} \tag{59}$$

Figure 8 shows the relative bearing capacity for the most critical mechanism shown in Figure 7.

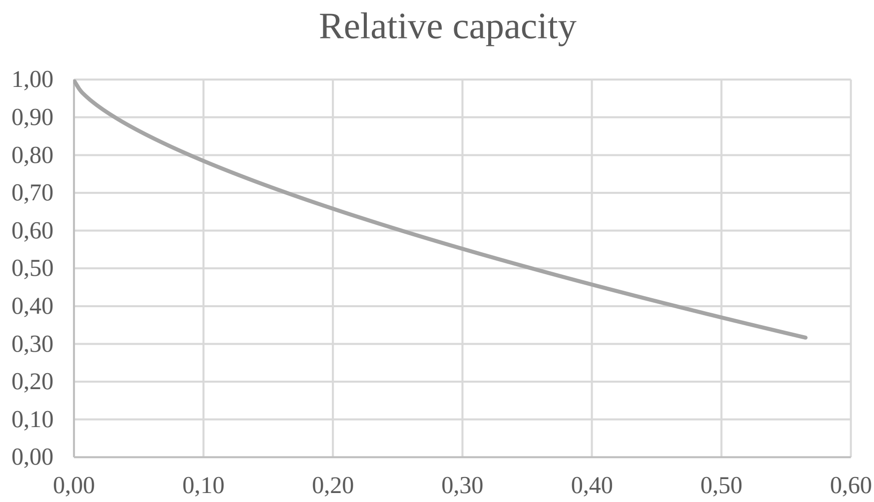


Figure 8 Relative capacity, that is $\frac{m_p}{\frac{P}{2\pi}}$ *as function of D/R₁*

Conclusion

A lower and upper bound solution for a circular foundation affected by a point load has been outlined. The same result is obtained with the two approaches, whereby this is shown to be an exact solution. With the lower bound solution, it is seen that m_θ varies parabolically with a maximum value at the center and zero at the edge, and that m_r is constantly equal to zero.

Next, a lower bound solution has been found for a square foundation affected by a centrally located point load. The form of the moment distribution in the square foundation is seen to be similar to the moment distribution on the circular foundation.

There is also an upper bound solution determined for a point foundation where the extent of the concentrated load is taken into account. With this approach, it can be seen that there is a possibility of a quite significant reduction of the dimensioning moment compared to the point foundation affected by a concentrated load without extension.

References

[1] Johansen, K. W.: Brudlinieteorier (Yield line theories), Copenhagen, Gjellerup, 1943.

[2] Nielsen, M. P. and L. C. Hoang: Limit Analysis and Concrete Plasticity. CRC Press LLC. 2010

Appendix 1. Checking equilibrium equations for point load on circular foundation

The equilibrium equations are used to check the equilibrium solution.

1. $v_r \cdot r = \frac{\partial(m_r \cdot r)}{\partial r} - \frac{\partial m_{r\theta}}{\partial \theta} - m_\theta$
2. $v_\theta = \frac{1}{r}\frac{\partial m_\theta}{\partial \theta} - \frac{\partial m_{r\theta}}{\partial r} - 2\frac{m_{r\theta}}{r}$
3. $\frac{\partial(v_r \cdot r)}{\partial r} + \frac{\partial v_\theta}{\partial \theta} = -p \cdot r$
4. $\frac{1}{r}\frac{\partial^2(m_r \cdot r)}{\partial r^2} - \frac{2}{r^2}\frac{\partial^2(m_{r\theta} \cdot r)}{\partial r \partial \theta} + \frac{1}{r^2}\frac{\partial^2 m_\theta}{\partial \theta^2} - \frac{1}{r}\frac{\partial m_\theta}{\partial r} = -p$

Due to rotational symmetry $v_\theta = m_{r\theta} = 0$.

As a boundary condition we have that $v_r = m_r = 0$ at the edge.

The shear force $v_r(r)$ is determined by
$v_r \cdot 2\pi r = -p \cdot \pi R^2 + p \cdot \pi r^2$
and $v_r \cdot r$ is given by:
$v_r \cdot r = -\frac{1}{2} p \cdot R^2 + \frac{1}{2} p \cdot r^2$
$v_r \cdot r = -\frac{1}{2} p \cdot R^2 \left(1 - \left(\frac{r}{R}\right)^2\right)$

With $P = \pi R^2 p$ inserted:

$v_r \cdot r = -\frac{P}{2\pi} \left(1 - \left(\frac{r}{R}\right)^2\right)$

1. equilibrium equation is used to determine m_r:
$v_r \cdot r = \frac{\partial(m_r \cdot r)}{\partial r} - \frac{\partial m_{r\theta}}{\partial \theta} - m_\theta$

Inserted:
$-\frac{P}{2\pi}\left(1 - \left(\frac{r}{R}\right)^2\right) = \frac{\partial(m_r \cdot r)}{\partial r} - 0 - \frac{P}{2\pi}\left(1 - \left(\frac{r}{R}\right)^2\right)$
$\frac{\partial(m_r \cdot r)}{\partial r} = 0$
$m_r \cdot r = c_1$
$m_r = \frac{c_1}{r}$

As m_r must provide final values for $r \to 0$ must apply $c_1 = 0$, and therefor:
$m_r = 0$

2. equilibrium equation is immediately seen to be fulfilled:
$v_\theta = \frac{1}{r}\frac{\partial m_\theta}{\partial \theta} - \frac{\partial m_{r\theta}}{\partial r} - 2\frac{m_{r\theta}}{r}$
$0 = \frac{1}{r} \cdot 0 - 0 - 2 \cdot 0$
$0 = 0$

3. equilibrium equation is seen to be satisfied:
$\frac{\partial(v_r \cdot r)}{\partial r} + \frac{\partial v_\theta}{\partial \theta} = -(-p) \cdot r$
$\frac{\partial(v_r \cdot r)}{\partial r} + \frac{\partial v_\theta}{\partial \theta} = p \cdot r$
$-\frac{1}{2} q \cdot R^2 \left(0 - 2\frac{r}{R^2}\right) = p \cdot r$
$p \cdot r = p \cdot r$

4. equilibrium equation is seen to be satisfied:
$\frac{1}{r}\frac{\partial^2(m_r \cdot r)}{\partial r^2} - \frac{2}{r^2}\frac{\partial^2(m_{r\theta} \cdot r)}{\partial r \partial \theta} + \frac{1}{r^2}\frac{\partial^2 m_\theta}{\partial \theta^2} - \frac{1}{r}\frac{\partial m_\theta}{\partial r} = -(-p)$
$\frac{1}{r} \cdot 0 - \frac{2}{r^2} \cdot 0 + \frac{1}{r^2} \cdot 0 - \frac{1}{r} \cdot \frac{P}{2\pi}\left(0 - \frac{2r}{R^2}\right) = p$
$\frac{P}{\pi R^2} = p$
$p = p$

The solution is thus seen to be in equilibrium. For a circular point foundation affected by a centrally located point load:

$m_\theta(r) = \frac{P}{2\pi}\left(1 - \left(\frac{r}{R}\right)^2\right)$
$m_r(r) = 0$
$v_r = -\frac{P}{2\pi r}\left(1 - \frac{r^2}{R^2}\right)$

Danish Society for Structural Science and Engineering

Requests for membership of the society are submitted to one of the board members:

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The purpose of the society is to work for the scientific development of structural mechanics - both theory and construction of all kinds of load-bearing structures - promote interest in the subject, work for a collegial relationship between its practitioners and assert its importance to and in collaboration with other branches of engineering. The purpose is sought realized through meetings with lectures and discussions as well as through the publication of the Proceedings of the Danish Society for Structural Science and Engineering.

Individual members, companies and institutions that are particularly interested in structural mechanics or whose company falls within the field of structural mechanics can be admitted as members.